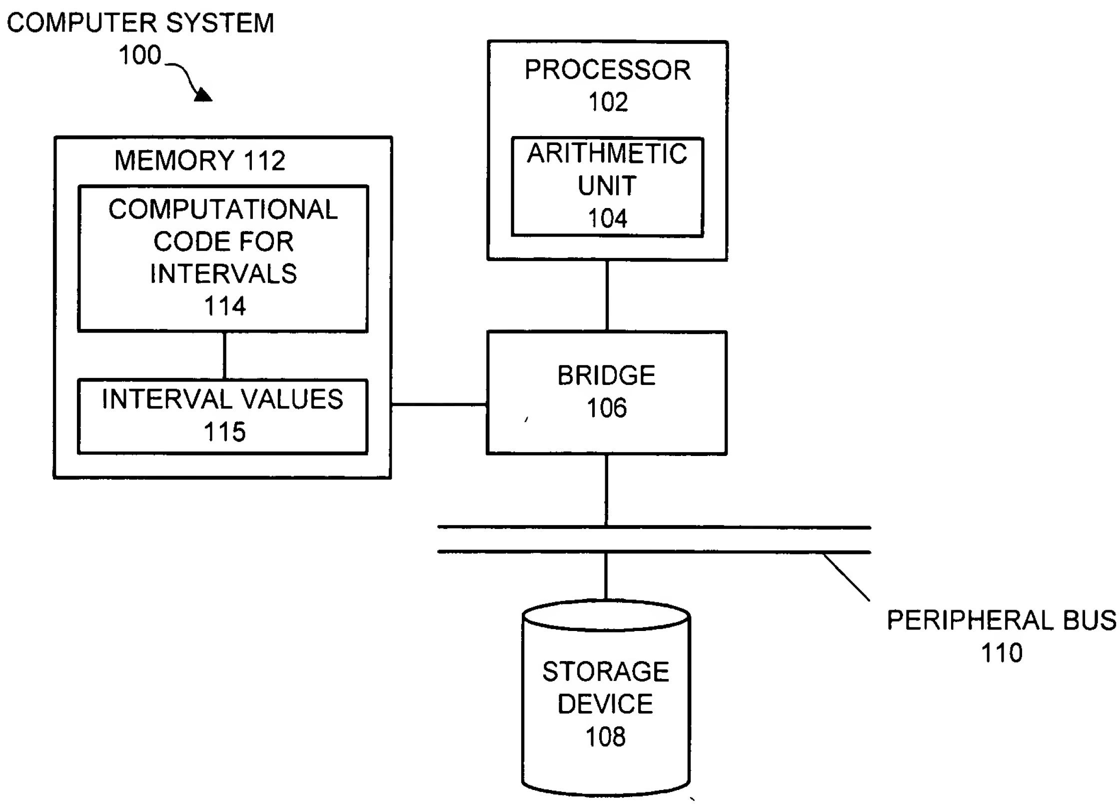

COMPUTER SYSTEM
100
MEMORY 112
COMPUTATIONAL
CODE FOR
INTERVALS
114
INTERVAL VALUES
115
PROCESSOR
102
ARITHMETIC
UNIT
104
BRIDGE
106
STORAGE
DEVICE
108
PERIPHERAL BUS
110

FIG. 1

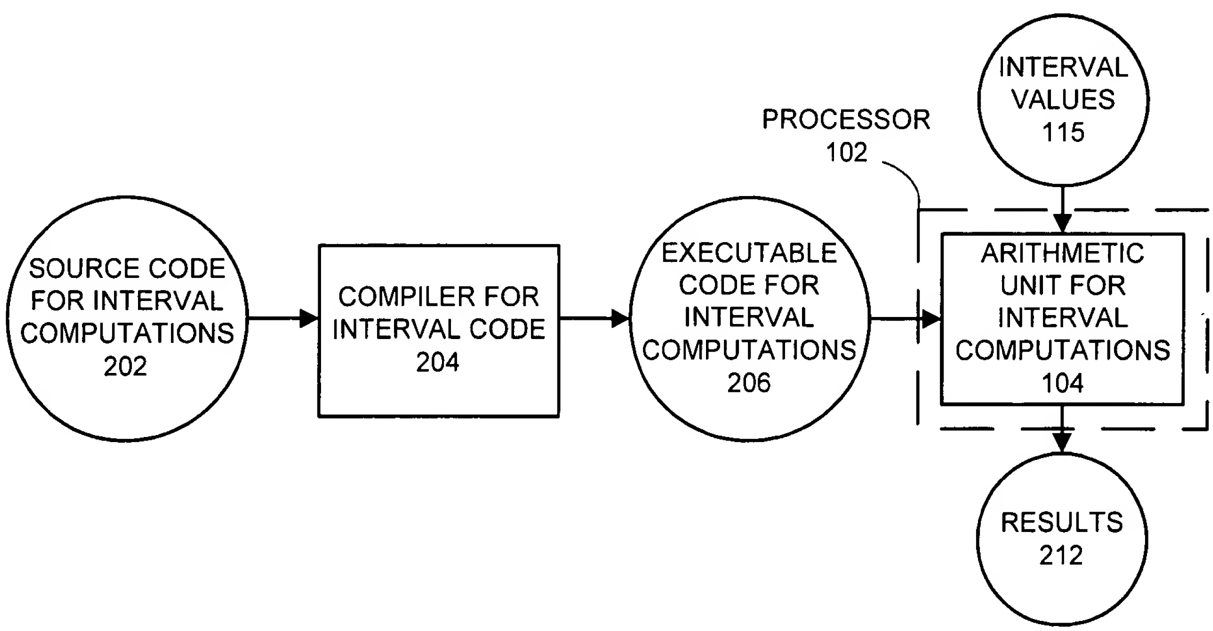
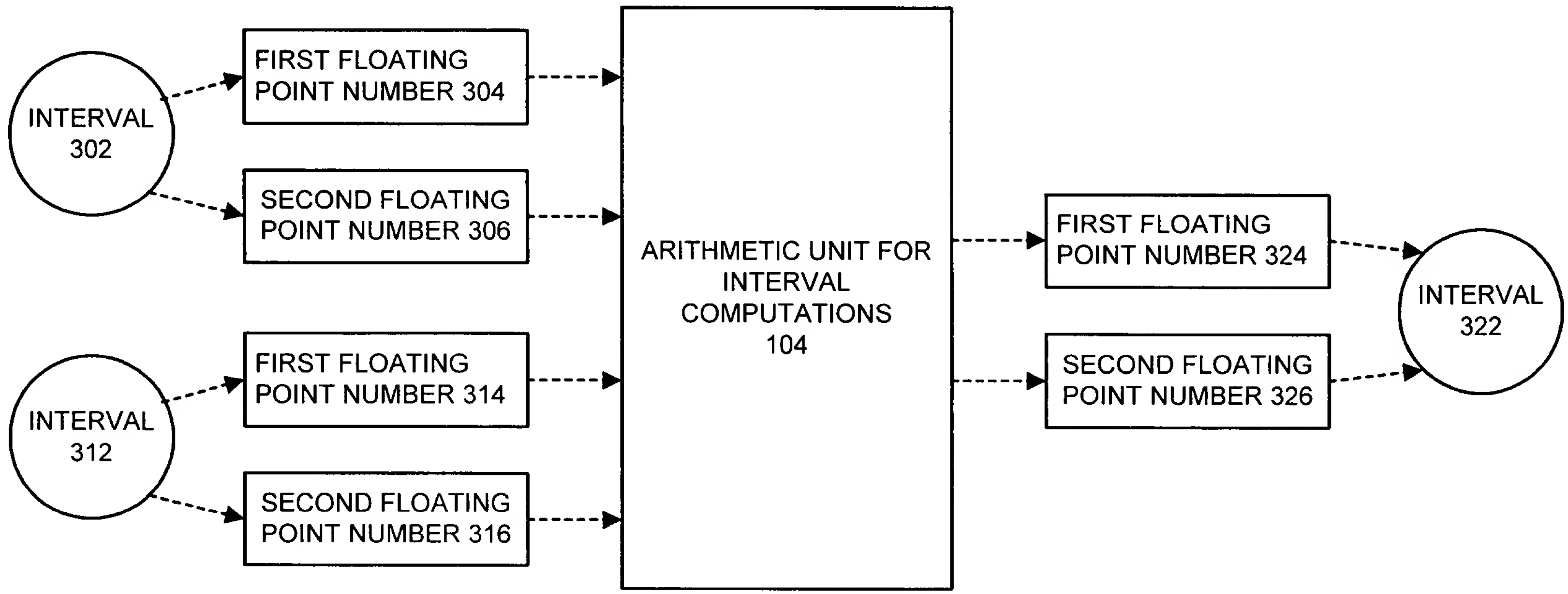
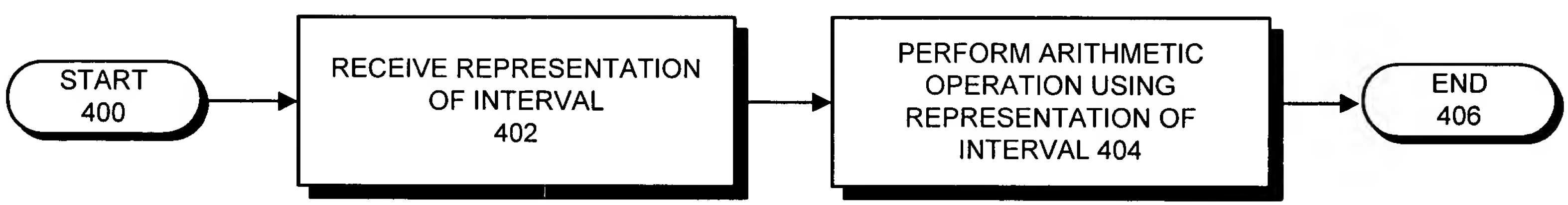

SOURCE CODE
FOR INTERVAL
COMPUTATIONS
202
COMPILER FOR
INTERVAL CODE
204
EXECUTABLE
CODE FOR
INTERVAL
COMPUTATIONS
206
PROCESSOR
102
INTERVAL
VALUES
115
ARITHMETIC
UNIT FOR
INTERVAL
COMPUTATIONS
104
RESULTS
212

FIG. 2



INTERVAL 302
FIRST FLOATING POINT NUMBER 304
SECOND FLOATING POINT NUMBER 306
INTERVAL 312
FIRST FLOATING POINT NUMBER 314
SECOND FLOATING POINT NUMBER 316
ARITHMETIC UNIT FOR INTERVAL COMPUTATIONS 104
FIRST FLOATING POINT NUMBER 324
SECOND FLOATING POINT NUMBER 326
INTERVAL 322

FIG. 3



START 400
RECEIVE REPRESENTATION OF INTERVAL 402
PERFORM ARITHMETIC OPERATION USING REPRESENTATION OF INTERVAL 404
END 406

FIG. 4

$$X \equiv [\underline{x}, \bar{x}] \equiv \{x \in \Re^* \mid \underline{x} \le x \le \bar{x}\}$$

$$Y \equiv [\underline{y}, \bar{y}] \equiv \{y \in \Re^* \mid \underline{y} \le y \le \bar{y}\}$$

(1) $X + Y = [\downarrow \underline{x} + \underline{y}, \uparrow \bar{x} + \bar{y}]$

(2) $X - Y = [\downarrow \underline{x} - \bar{y}, \uparrow \bar{x} - \underline{y}]$

(3) $X \times Y = [\min(\downarrow \underline{x} \times \underline{y}, \underline{x} \times \bar{y}, \bar{x} \times \underline{y}, \bar{x} \times \bar{y}), \max(\uparrow \underline{x} \times \underline{y}, \underline{x} \times \bar{y}, \bar{x} \times \underline{y}, \bar{x} \times \bar{y})]$

(4) $X / Y = [\min(\downarrow \underline{x} / \underline{y}, \underline{x} / \bar{y}, \bar{x} / \underline{y}, \bar{x} / \bar{y}), \max(\uparrow \underline{x} / \underline{y}, \underline{x} / \bar{y}, \bar{x} / \underline{y}, \bar{x} / \bar{y})]$, if $0 \notin Y$

$X / Y \subseteq \Re^*$, if $0 \in Y$

FIG. 5

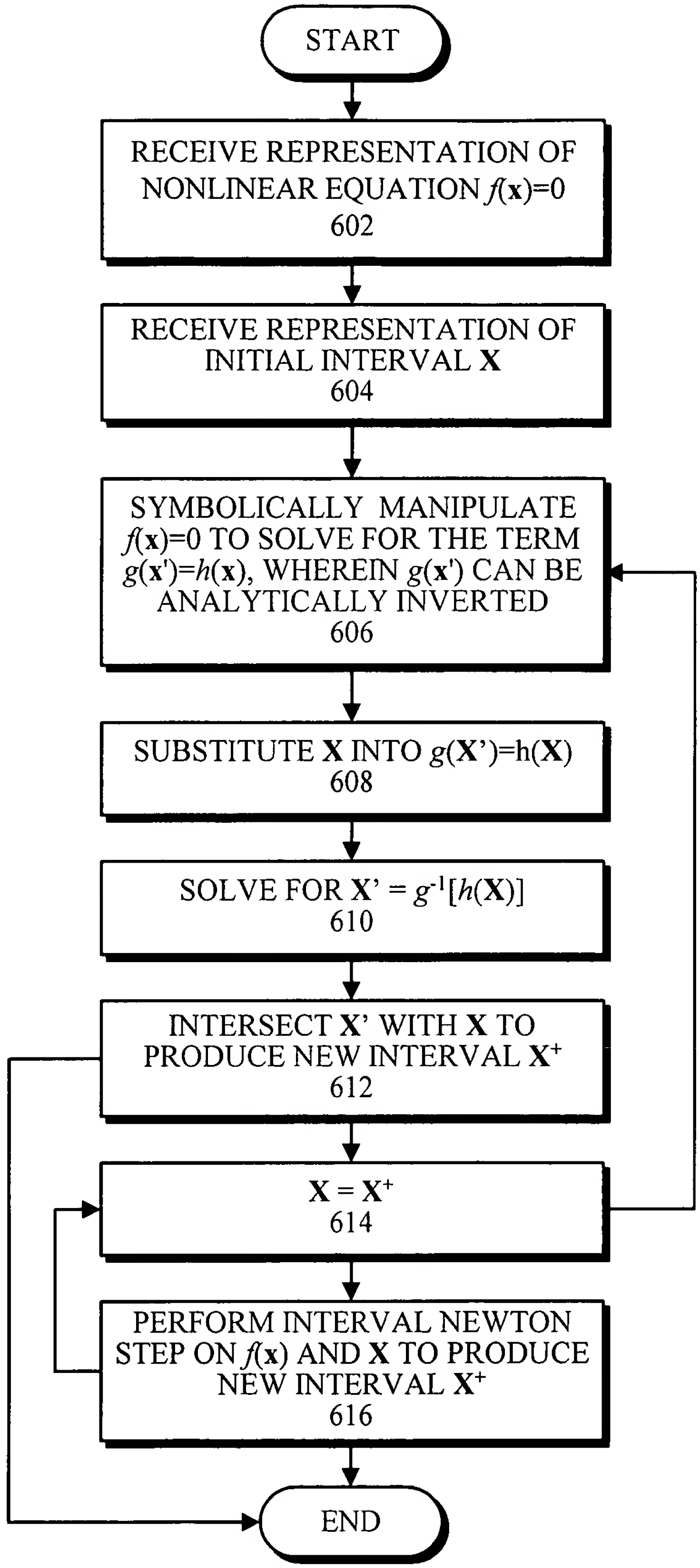

START
RECEIVE REPRESENTATION OF NONLINEAR EQUATION $f(\mathbf{x})=0$
602
RECEIVE REPRESENTATION OF INITIAL INTERVAL $\mathbf{X}$
604
SYMBOLICALLY MANIPULATE $f(\mathbf{x})=0$ TO SOLVE FOR THE TERM $g(\mathbf{x}')=h(\mathbf{x})$, WHEREIN $g(\mathbf{x}')$ CAN BE ANALYTICALLY INVERTED
606
SUBSTITUTE $\mathbf{X}$ INTO $g(\mathbf{X}')=\mathrm{h}(\mathbf{X})$
608
SOLVE FOR $\mathbf{X}' = g^{-1}[h(\mathbf{X})]$
610
INTERSECT $\mathbf{X}'$ WITH $\mathbf{X}$ TO PRODUCE NEW INTERVAL $\mathbf{X}^+$
612
$\mathbf{X} = \mathbf{X}^+$
614
PERFORM INTERVAL NEWTON STEP ON $f(\mathbf{x})$ AND $\mathbf{X}$ TO PRODUCE NEW INTERVAL $\mathbf{X}^+$
616
END

FIG. 6

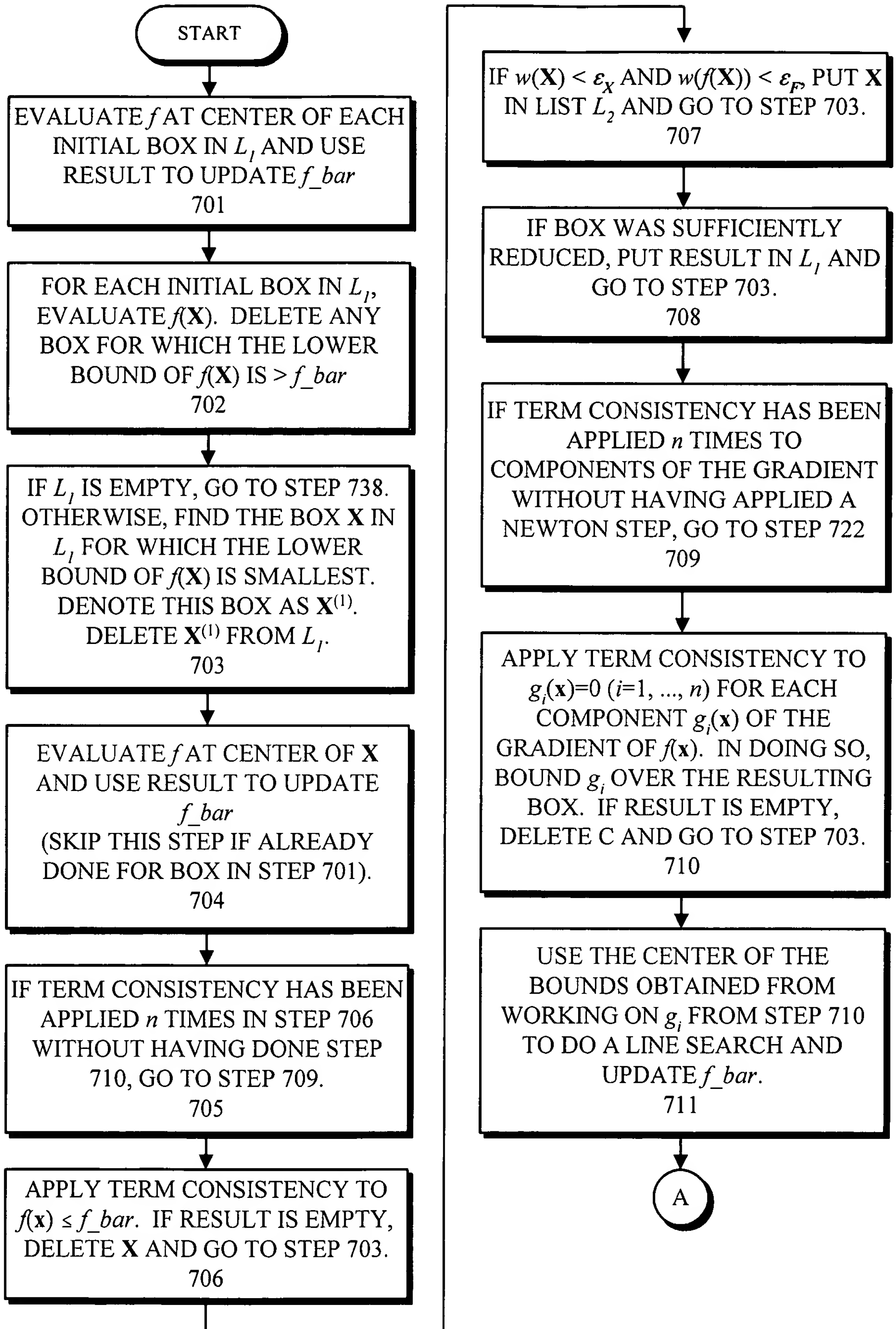

START
EVALUATE f AT CENTER OF EACH INITIAL BOX IN L_1 AND USE RESULT TO UPDATE f_bar
701
FOR EACH INITIAL BOX IN L_1, EVALUATE $f(\mathbf{X})$. DELETE ANY BOX FOR WHICH THE LOWER BOUND OF $f(\mathbf{X})$ IS > f_bar
702
IF L_1 IS EMPTY, GO TO STEP 738. OTHERWISE, FIND THE BOX $\mathbf{X}$ IN L_1 FOR WHICH THE LOWER BOUND OF $f(\mathbf{X})$ IS SMALLEST. DENOTE THIS BOX AS $\mathbf{X}^{(1)}$. DELETE $\mathbf{X}^{(1)}$ FROM L_1.
703
EVALUATE f AT CENTER OF $\mathbf{X}$ AND USE RESULT TO UPDATE f_bar (SKIP THIS STEP IF ALREADY DONE FOR BOX IN STEP 701).
704
IF TERM CONSISTENCY HAS BEEN APPLIED n TIMES IN STEP 706 WITHOUT HAVING DONE STEP 710, GO TO STEP 709.
705
APPLY TERM CONSISTENCY TO $f(\mathbf{x}) \le f\_bar$. IF RESULT IS EMPTY, DELETE $\mathbf{X}$ AND GO TO STEP 703.
706
IF $w(\mathbf{X}) < \varepsilon_X$ AND $w(f(\mathbf{X})) < \varepsilon_F$, PUT $\mathbf{X}$ IN LIST L_2 AND GO TO STEP 703.
707
IF BOX WAS SUFFICIENTLY REDUCED, PUT RESULT IN L_1 AND GO TO STEP 703.
708
IF TERM CONSISTENCY HAS BEEN APPLIED n TIMES TO COMPONENTS OF THE GRADIENT WITHOUT HAVING APPLIED A NEWTON STEP, GO TO STEP 722
709
APPLY TERM CONSISTENCY TO $g_i(\mathbf{x})=0$ ($i=1, ..., n$) FOR EACH COMPONENT $g_i(\mathbf{x})$ OF THE GRADIENT OF $f(\mathbf{x})$. IN DOING SO, BOUND g_i OVER THE RESULTING BOX. IF RESULT IS EMPTY, DELETE C AND GO TO STEP 703.
710
USE THE CENTER OF THE BOUNDS OBTAINED FROM WORKING ON g_i FROM STEP 710 TO DO A LINE SEARCH AND UPDATE f_bar.
711
A

FIG. 7A

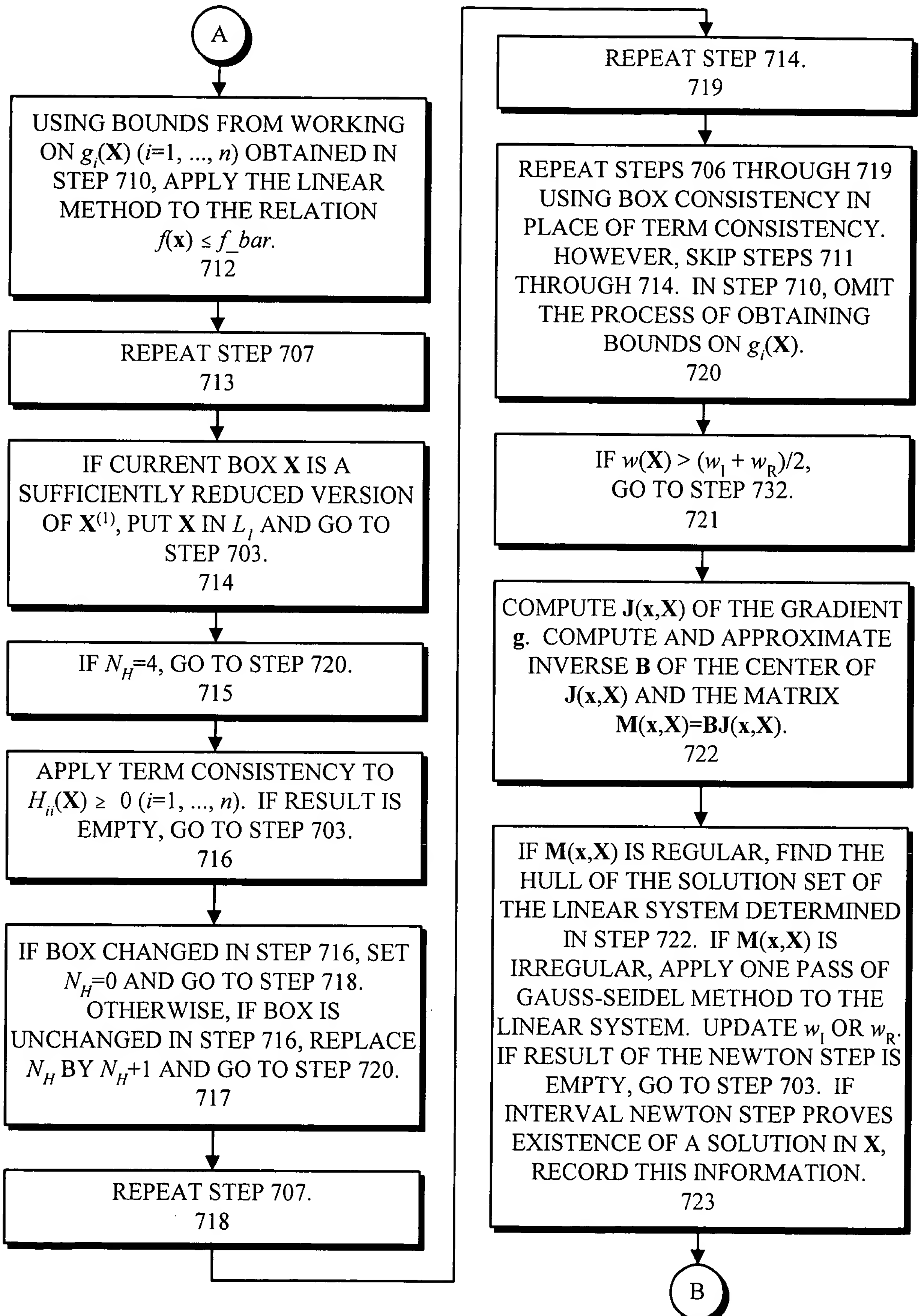

A
USING BOUNDS FROM WORKING ON $g_i(\mathbf{X})$ (i=1, ..., n) OBTAINED IN STEP 710, APPLY THE LINEAR METHOD TO THE RELATION $f(\mathbf{x}) \le f\_bar$. 712
REPEAT STEP 707 713
IF CURRENT BOX **X** IS A SUFFICIENTLY REDUCED VERSION OF $\mathbf{X}^{(1)}$, PUT **X** IN L_l AND GO TO STEP 703. 714
IF N_H=4, GO TO STEP 720. 715
APPLY TERM CONSISTENCY TO $H_{ii}(\mathbf{X}) \ge 0$ (i=1, ..., n). IF RESULT IS EMPTY, GO TO STEP 703. 716
IF BOX CHANGED IN STEP 716, SET N_H=0 AND GO TO STEP 718. OTHERWISE, IF BOX IS UNCHANGED IN STEP 716, REPLACE N_H BY N_H+1 AND GO TO STEP 720. 717
REPEAT STEP 707. 718
REPEAT STEP 714. 719
REPEAT STEPS 706 THROUGH 719 USING BOX CONSISTENCY IN PLACE OF TERM CONSISTENCY. HOWEVER, SKIP STEPS 711 THROUGH 714. IN STEP 710, OMIT THE PROCESS OF OBTAINING BOUNDS ON $g_i(\mathbf{X})$. 720
IF $w(\mathbf{X}) > (w_I + w_R)/2$, GO TO STEP 732. 721
COMPUTE **J(x,X)** OF THE GRADIENT **g**. COMPUTE AND APPROXIMATE INVERSE **B** OF THE CENTER OF **J(x,X)** AND THE MATRIX **M(x,X)=BJ(x,X)**. 722
IF **M(x,X)** IS REGULAR, FIND THE HULL OF THE SOLUTION SET OF THE LINEAR SYSTEM DETERMINED IN STEP 722. IF **M(x,X)** IS IRREGULAR, APPLY ONE PASS OF GAUSS-SEIDEL METHOD TO THE LINEAR SYSTEM. UPDATE w_I OR w_R. IF RESULT OF THE NEWTON STEP IS EMPTY, GO TO STEP 703. IF INTERVAL NEWTON STEP PROVES EXISTENCE OF A SOLUTION IN **X**, RECORD THIS INFORMATION. 723
B

FIG. 7B

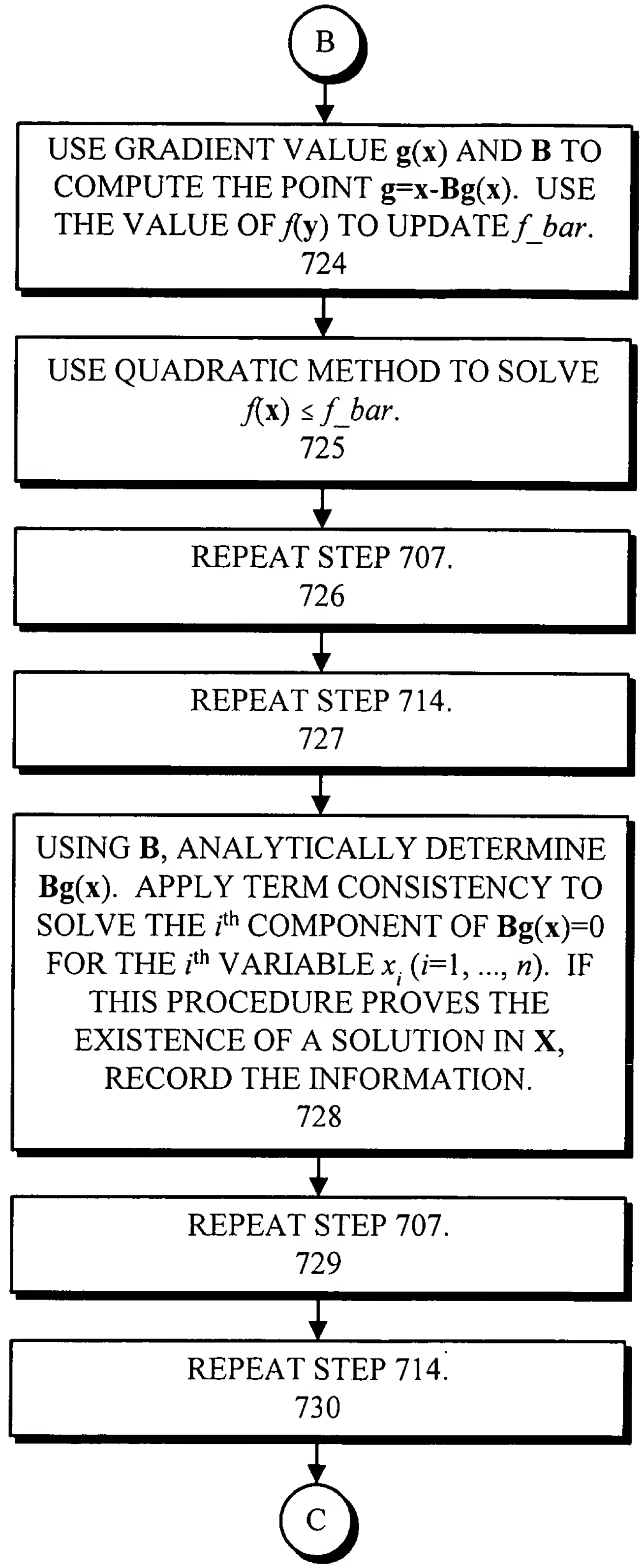

B
USE GRADIENT VALUE g(x) AND B TO COMPUTE THE POINT g=x-Bg(x). USE THE VALUE OF f(y) TO UPDATE f_bar.
724
USE QUADRATIC METHOD TO SOLVE f(x) ≤ f_bar.
725
REPEAT STEP 707.
726
REPEAT STEP 714.
727
USING B, ANALYTICALLY DETERMINE Bg(x). APPLY TERM CONSISTENCY TO SOLVE THE ith COMPONENT OF Bg(x)=0 FOR THE ith VARIABLE xi (i=1, ..., n). IF THIS PROCEDURE PROVES THE EXISTENCE OF A SOLUTION IN X, RECORD THE INFORMATION.
728
REPEAT STEP 707.
729
REPEAT STEP 714.
730
C

FIG. 7C

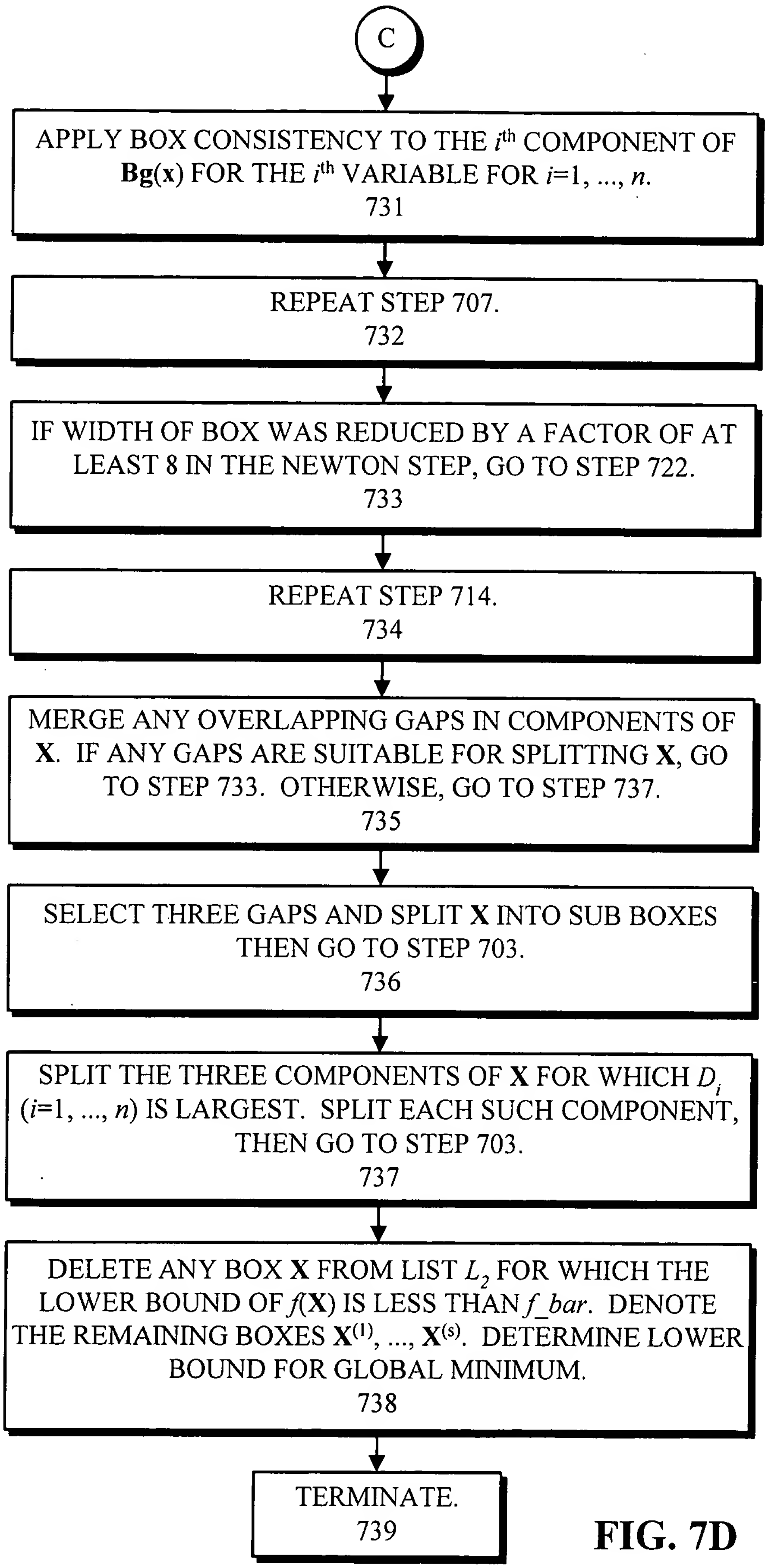

C
APPLY BOX CONSISTENCY TO THE i^{th} COMPONENT OF **Bg**(**x**) FOR THE i^{th} VARIABLE FOR i=1, ..., n.
731
REPEAT STEP 707.
732
IF WIDTH OF BOX WAS REDUCED BY A FACTOR OF AT LEAST 8 IN THE NEWTON STEP, GO TO STEP 722.
733
REPEAT STEP 714.
734
MERGE ANY OVERLAPPING GAPS IN COMPONENTS OF **X**. IF ANY GAPS ARE SUITABLE FOR SPLITTING **X**, GO TO STEP 733. OTHERWISE, GO TO STEP 737.
735
SELECT THREE GAPS AND SPLIT **X** INTO SUB BOXES THEN GO TO STEP 703.
736
SPLIT THE THREE COMPONENTS OF **X** FOR WHICH D_i (i=1, ..., n) IS LARGEST. SPLIT EACH SUCH COMPONENT, THEN GO TO STEP 703.
737
DELETE ANY BOX **X** FROM LIST L_2 FOR WHICH THE LOWER BOUND OF f(**X**) IS LESS THAN f_bar. DENOTE THE REMAINING BOXES $\mathbf{X}^{(1)}, ..., \mathbf{X}^{(s)}$. DETERMINE LOWER BOUND FOR GLOBAL MINIMUM.
738
TERMINATE.
739

FIG. 7D